647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2024 (Old Course)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one: (07)

(1) Define Equivalence relation. Show that similarity of matrices on the set of $n \times n$ matrices is an equivalence relation.

(2) Define Direct product of two lattice. Prove that direct product of two lattices is also a lattice.

Que-1(B) Answer any one: (04)

(1) Define Lattice as an algebraic structure. Let A be a non-empty set and P(A) be the power set of A. For $X, Y \in P(A)$, $X * Y = X \cap Y, X \oplus Y = X \cup Y$ then prove that $(L, *, \oplus)$ is a lattice.

(2) Define Complemented lattice. Give an example of a bounded lattice which is not a complemented lattice.

Que-1(C) Answer any one: (03)

(1) Define Isomorphism of two lattices. If L_1 is the lattice $S_6 = \{1,2,3,6\}$ and L_2 is the lattice $(P(S), \subseteq)$ whrere $S = \{a, b\}$ then show that S_6 is isomorphic to $P(S)$.

(2) Define: Lower and upper bound in poset.
Lower and upper bound of poset $(\{1,2,3,4,6,12\}, /)$ are _____ and _______

Que-2(A) Answer any one: (07)

(1) State and prove D'Morgan's law for the Boolean algebra $(B, *, \oplus, ', 0, 1)$

(2) State and prove Stone's representation theorem in Boolean algebra

Que-2(B) Answer any one: (04)

(1) Define Minterms in n-variables. Prove that the sum of all minterms of n-variables $x_1, x_2, \ldots x_n$ is 1.

(2) Define Cube array representation of Boolean function. Find cube array representation of $f(x, y, z) = xy + xz'$.

Que-2(C) Answer any one: (03)

(1) Prove that a non-zero element a of Boolean algebra $(B, *, \oplus, ', 0, 1)$ is an atom iff $\forall\, x \in B$ either $a * x = 0$ or $a * x = a$.

(2) If m_i and m_j are distinct minterms in n-variables $x_1, x_2, \ldots x_n$ then prove that $m_i * m_j = 0$.

Que-3(A) Answer any one. (07)

(1) Obtain Cauchy-Riemann conditions for an analytic function $f(z)$ in polar form.

(2) Prove that $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & ; \quad z \neq 0 \\ 0 & ; \quad z = 0 \end{cases}$ satisfies C-R conditions at origin, however $f(z)$ is not analytic at origin.

Que-3(B) Answer any one. (04)

(1) Define Harmonic Function and show that $u = \frac{1}{2}log(x^2 + y^2)$ is harmonic function and find its conjugate harmonic function.

(2) Prove that an analytic function of a constant modulus is also constant in its domain.

Que-3(C) Answer any one. (03)

(1) Prove that $u = \left(r - \frac{1}{r}\right) sin\theta$ is harmonic function.

(2) If $f(z) = x^2 + ayx + by^2 + i(x^2 + cxy + dy^2)$ is analytic then find value of a, b, c & d.

Que-4(A) Answer any one. (07)

(1) State and prove Cauchy's integral formula.

(2) State Green's theorem and state and prove Cauchy's fundamental theorem.

Que-4(B) Answer any one. (04)

(1) In usual notation prove that $\left|\int_a^b f(z)dz\right| \leq \int_a^b |f(z)|\, dz$.

(2) Show that $\left|\int_c \frac{z+4}{z^3-1} dz\right| \leq \frac{6\pi}{7}$, where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$.

Que-4(C) Answer any one. (03)

(1) Find $\int_c f(z)dz$, where $f(z) = y^2$; $y > 1$ and $f(z) = 1 - y$; $y < 1$ and C is a part of curve $x = y^3$ from $z = 1$ to $1 + 2i$.

(2) Find $\int_c \frac{z}{(9-z^2)(z+i)} dz, C: |z| = 2$.

Que-5(A) Answer any one. (07)

(1) State and prove Cauchy's integral formula for derivatives and deduce that $f^n(z_0) = \frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz$.

(2) State Cauchy's inequality and state and prove Morera's theorem.

Que-5(B) Answer any one. (04)

(1) State Cauchy's integral formula for derivatives and if $g(z_0) = \int_c \frac{z^2-2z+5}{z-z_0} dz, C: |z - 1| = 2$ then find $g(i), g(0), g(2), g'(2), g(10 + i)$.

(2) Evaluate $\int_c \frac{z^2+3}{z^2(z-4)} dz$, $C : |z| = 1$

Que-5(C) Answer any one. (03)

(1) Find $\int_c \frac{sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, $C : |z| = 1$

(2) Evaluate: $\int_c \frac{5z^2-7z+3}{(z-1)^2} dz$, $C : |z| = 2$

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